Stable Marriage Problem

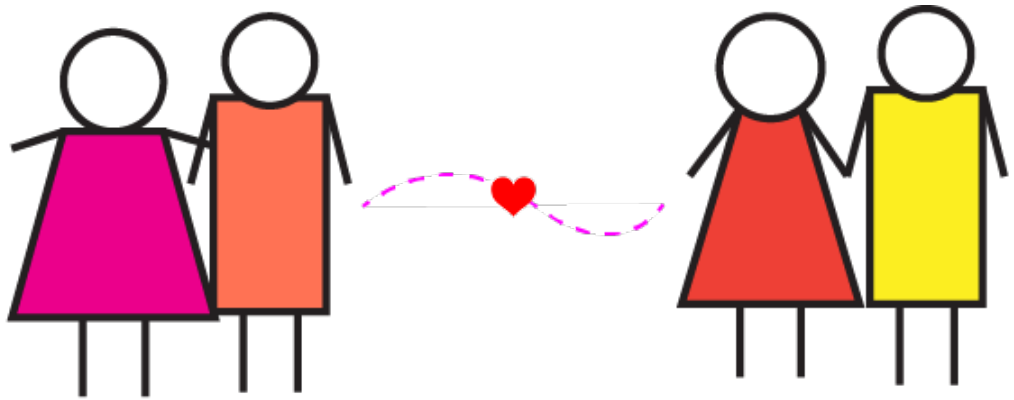
Fernando Martínez

Stable + Marriage

these two words combined, is something everybody wants, but it is not always possible.

You need to choose what you want, but also you must be chosen!

Interactions where you care with whom you are dealing

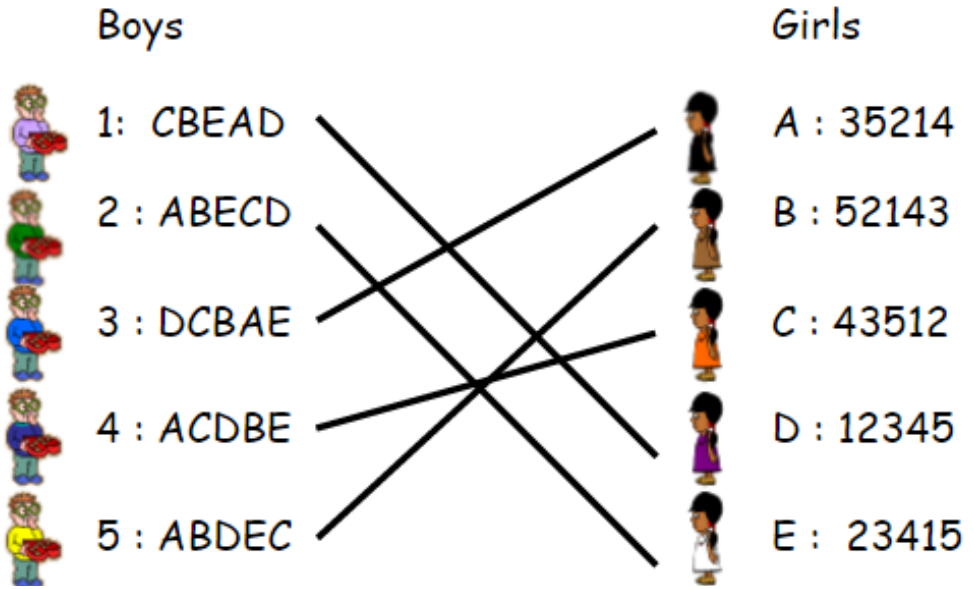


Motivation

The Stable Marriage Problem is a mathematical/game theory problem.

Applied in multiple areas/disciplines:

- **Medicine:** Match organ donors with patients, saving many lives every year;
- **Labor Market:** Match employees with employers;
- **Education**: Match students with schools/universities and medicine student residents with hospitals;
- **Computer Science:** In a large distributed internet service, assigning users to servers.



Exampling the Problem

We have two equally-sized sets of people (n-elements), **Men** and **Women**, and we seek to **match** them where pairs must be formed of **one** element from **Men** and one from **Women**.

A matching is **stable** if no unmatched man and woman each prefers any other to his or her spouse.

(There is no room for impossible loves!)

To solve this problem, we use the **Gale-Shapley algorithm!**

Exampling the Problem – Gale-Shapley Alg.

The **Gale-Shapley algorithm** consists of numerous iterations, as illustrated by the following:

- Day 0: Everyone rates the opposing sex.
- Day 1: Each man proposes to his top choice. Consecutively each woman rejects all but her top suitor.
- Day *n* + 1: Each man rejected on day *n* proposes to his top remaining pick. Then each woman rejects everyone but her top suitor.
- The algorithm is completed when **everyone is engaged**.

Exampling the Problem – Gale-Shapley Alg.

Men	Men's Ranking		
John	Tori	Michelle	Mary
Lucas	Mary	Michelle	Tori
Peter	Mary	Tori	Michelle

Women	Women's Ranking		
Mary	John	Lucas	Peter
Tori	Lucas	Peter	John
Michelle	Peter	Lucas	John

Exampling the Problem – Gale-Shapley Alg.
Results after running the algorithm on our Example

Men	Men's Ranking		
John	Tori	Michelle	Mary
Lucas	Mary	Michelle	Tori
Peter	Mary	Tori	Michelle

Women	Women's Ranking		
Mary	John	Lucas	Peter
Tori	Lucas	Peter	John
Michelle	Peter	Lucas	John

WE HAVE A MATCH!!

(Michelle, John), (Mary, Lucas), (Peter, Tori)

Gale-Shapley Algorithm Pseudocode

1. **GALES_HAPLEY**(Men, Women):
2. **while** there is an unmarried *man* **do**
3. *man* chooses the first *woman* on his preference list he has not proposed to yet and proposes to her
4. **if** *woman* is unmarried or prefers *man* over her current partner man^0 **then**
5. *woman* divorces man^0
6. *woman* marries *man*

Time Complexity: $O(N^2)$

Key Takeaway

All members on the left side (who propose) will be assigned **the best potential mate from the stable matchings** (no trade-off). The result is the opposite for those on the right (who respond); everyone gets the worst possible outcome.

This does not imply that they all receive their least favorite partner but that **they get the one with their least preferred partner out of all possible stable matches**.

Real Life Complications

Real life complications

Variations